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Correction to "Self-similar behavior of plasma fluid equations – II"*

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Equation (14) should be changed to read (i.e. $\delta \not\equiv 0$),

$$X = \mu + vx + \delta t, \;\; T = \rho + vt, \;\; U = -2v\mu, \;\; V = \delta, \;\; W = -2vw. \tag{14}$$

The resulting invariants for various values of the constants μ, v, ρ and δ are listed below along with the resulting ordinary differential equations (ODE).

CASE I: $\mu \neq 0$, $v \neq 0$, $\rho \neq 0$, $\delta \neq 0$
The invariants are:

$$I_1 \equiv \xi = \frac{x + \frac{\mu}{v}}{t + \frac{\rho}{v}} - \frac{\delta}{v}\left[\ln\left(t + \frac{\rho}{v}\right) + \frac{\frac{\rho}{v}}{t + \frac{\rho}{v}}\right],$$

$$I_2 \equiv N_i(\xi) = \left(t + \frac{\rho}{v}\right)^2 n_i(x, t), \tag{I-A}$$

$$I_3 \equiv V_i(\xi) = v_i(x, t) - \frac{\delta}{v}\ln\left(t + \frac{\rho}{v}\right),$$

$$I_4 \equiv N_e(\xi) = \left(t + \frac{\rho}{v}\right)^2 n_e(x, t).$$

The ODE are:

$$-2N_i - \frac{\xi\, dN_i}{d\xi} + \frac{d(N_i V_i)}{d\xi} - \frac{\delta}{v}\frac{dN_i}{d\xi} = 0,$$

$$(V_i - \xi)\frac{dV_i}{d\xi} + \frac{1}{N_e}\frac{dN_e}{d\xi} - \frac{\delta}{v}\left(\frac{dV_i}{d\xi} - 1\right) = 0, \tag{I-B}$$

$$\frac{d}{d\xi}\left(\frac{1}{N_e}\frac{dN_e}{d\xi}\right) = N_e - N_i,$$

CASE II: $\mu \neq 0,\ \nu \neq 0,\ \rho \neq 0,\ \delta = 0$
The invariants are:

$$I_1 \equiv \xi = \frac{x + \dfrac{\mu}{\nu}}{t + \dfrac{\rho}{\nu}},\ I_2 \equiv N_i(\xi) = \left(t + \frac{\rho}{\nu}\right)^2 n_i(x, t),$$

$$I_3 \equiv V_i(\xi) = v_i(x, t),\ I_4 \equiv N_e(\xi) = \left(t + \frac{\rho}{\nu}\right)^2 n_e(x, t), \tag{II-A}$$

The ODE are:

$$-2N_i - \frac{\xi\, dN_i}{d\xi} + \frac{d(N_i V_i)}{d\xi} = 0,$$

$$(V_i - \xi)\frac{dV_i}{d\xi} + \frac{1}{N_e}\frac{dN_e}{d\xi} = 0, \tag{II-B}$$

$$\frac{d}{d\xi}\left(\frac{1}{N_e}\frac{dN_e}{d\xi}\right) = N_e - N_i.$$

CASE III: $\mu = 0,\ \nu = 0,\ \rho \neq 0,\ \delta \neq 0$
The invariants are:

$$I_1 \equiv \xi = \rho x - \frac{\delta t^2}{2},\ I_2 \equiv N_i(\xi) = n_i(x, t), \tag{III-A}$$

$$I_3 \equiv V_i(\xi) = \rho v_i(x, t) - \delta t,\ I_4 \equiv N_e(\xi) = n_e(x, t).$$

The ODE are:

$$\frac{d}{d\xi}(N_i V_i) = 0,\ \delta + V_i\frac{dV_i}{d\xi} + \frac{\rho^2}{N_e}\frac{dN_e}{d\xi} = 0, \tag{III-B}$$

$$\rho^2\frac{d}{d\xi}\left(\frac{1}{N_e}\frac{dN_e}{d\xi}\right) = N_e - N_i.$$

CASE IV: $\mu = 0,\ \nu = 0,\ \rho = 0,\ \delta \neq 0$
The invariants are:

$$I_1 = \xi = t,\ I_2 \equiv N_i(\xi) = n_i(x, t),$$

$$I_3 \equiv V_i(\xi) = \frac{x}{t} - v_i(x, t),\ I_4 \equiv N_e(\xi) = n_e(x, t). \tag{IV-A}$$

The ODE are:

$$\frac{dN_i}{d\xi} = 0,\ \frac{dV_i}{d\xi} + \frac{V_i}{\xi} = 0,\ N_e = N_i. \tag{IV-B}$$

CASE V: $\mu \neq 0,\ \nu = 0,\ \rho \neq 0,\ \delta = 0$
The invariants are:

$$I_1 \equiv \xi = x - \frac{\mu}{\rho} t,\ I_2 \equiv N_i(\xi) = n_i(x, t),$$
$$I_3 \equiv V_i(\xi) = v_i(x, t),\ I_4 \equiv N_e(\xi) = n_e(x, t). \tag{V-A}$$

The ODE are:

$$-\frac{\mu}{\rho}\frac{dN_i}{d\xi} + \frac{d(N_i V_i)}{d\xi} = 0,\quad -\frac{\mu}{\rho}\frac{dV_i}{d\xi} + V_i\frac{dV_i}{d\xi} + \frac{1}{N_e}\frac{dN_e}{d\xi} = 0,$$
$$\frac{d}{d\xi}\left(\frac{1}{N_e}\frac{dN_e}{d\xi}\right) = N_e - N_i \tag{V-B}$$

Cases II and V were discussed by us (Shen and Lonngren [1]) and Cases III and IV were discussed by Zhmudsky (Zhmudsky [2]). Case I has not been noted previously.

REFERENCES

[1] H. Shen and K. E. Lonngren, Self-similar behavior of plasma fluid equations – II, *J. Engineering Math*, 10 (1976), 135–141.
[2] A. A. Zhmudsky, On the symmetry of plasma slow motion, *Ukrainskii Fizicheskii Zhurnal*, 3 (1975) 492–496.

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